CHAPTER

3

VARIATIONAL METHODS FOR IMAGE RESTORATION

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3.1 INTRODUCTION

The degradation of images caused by noise, blur, and other artefacts remains a key challenge in computer vision applications. In this chapter, a variational mathematical model for image restoration is introduced, framing the restoration process as an optimisation problem. The discussion explores how partial differential equations and regularisation techniques can help reconstruct degraded images while preserving important structural and textural features. Furthermore, the proposed method demonstrates improved performance across applications as diverse as medical imaging and historical photograph enhancement, producing higher-quality results than conventional filtering approaches.

Image processing can be seen as a subfield of a more classical field of signal processing, where signal processing techniques such as segmentation, restoration, compression, and reconstruction are applied to digital images (two-dimensional [2D] signals). One of the fundamental themes in image processing is image restoration. It is concerned with estimating a clean image from its corrupted observation. There is no doubt that corruption can occur in signals due to various factors; hence, digital images are also prone to corruption. In digital images, common corruptions are blur, noise, and a combination of both. Blur that occurs in images is usually the result of abrupt movements during the image acquisition process, the lens being out of focus, and image acquisition through a turbulent medium. Noise, in digital images, is another source of corruption.

Noise corruption in images usually follows some probability distribution. For instance, the most common assumption is the Gaussian distribution. In this case, the noise in the image follows an additive zero-mean, independent, and identically distributed Gaussian distribution. However, other noise distributions may also degrade the image, such as the Poisson distribution, the Cauchy distribution, and the Laplace distribution. To make things worse, corruption or degradation in images can be a mix of both noise and blur, thereby rendering specialised algorithms for the clean image estimation.

Restoration of images can be done by classical filtering methods such as mean, median, and Wiener filtering (Aguilar-González et al., 2014). These filters are simple to implement and, in some sense, effective for low levels of noise or blur. However, for more complex corruptions that require the output of a filtered image to retain sharpness, these filters might perform badly.

A more modern approach to image restoration is by variational methods, which involve mathematical optimisation. Usually, this approach seeks to minimise a cost function using a mathematical optimisation algorithm. Obtaining this minimum will result in obtaining the filtered image. In the preceding sections, this approach to image restoration will be discussed and explained more precisely.

3.2 UNDERSTANDING THE BASICS

In image restoration, the corruption of a digital image is given by the following degradation model:

$$\boldsymbol{f} = \boldsymbol{K}\boldsymbol{u} + \boldsymbol{n} \tag{3.1}$$

where $\boldsymbol{f} \in \mathbb{R}^{\boldsymbol{n}}$ is the corrupted image (observation), $\boldsymbol{K} \in \mathbb{R}^{\boldsymbol{m}\times\boldsymbol{n}}$ is a linear operator, $\boldsymbol{u} \in \mathbb{R}^{\boldsymbol{n}}$ is the clean image to be estimated, and $\boldsymbol{n} \in \mathbb{R}^{\boldsymbol{n}}$

is the additive noise. By 'classical' is meant those filtering algorithms learned in any standard text on image processing. This statement can be argued. This is because the mathematical techniques in variational analysis and mathematical optimisation are quite old. However, due to the explosion of compressed sensing (Donoho, 2006) theory, which bears some similarities in its mathematical model for sparse signal recovery and convex optimisation algorithms, image restoration following this paradigm has also increased.

The linear operator $\boldsymbol{K}$ is usually a convolution kernel that models blur. In some signal processing applications, such as reconstruction, this linear operator also models a projection. However, in this chapter, the discussion will only be when $\boldsymbol{K}$ is a blurring operator (convolution kernel) such as the Gaussian blur or box blur.

Additionally, this chapter also considers the situation when $\boldsymbol{K} = \boldsymbol{I}$, where $\boldsymbol{I}$ is the identity matrix. In this case, Equation (3.1) is a pure noise degradation model:

$$\boldsymbol{f} = \boldsymbol{u} + \boldsymbol{n} \tag{3.2}$$

because the corruption is only contributed by the additive noise $\boldsymbol{n}$. On the other hand, if the noise $\boldsymbol{n}$ is not present then,

$$\boldsymbol{f} = \boldsymbol{K}\boldsymbol{u} \tag{3.3}$$

is a pure blurring model without noise. It is tempting to use direct approaches to solve for the clean image $\boldsymbol{u}$ in both Equations (3.2) and (3.3). For example, from Equation (3.2)

$$\boldsymbol{u} = \boldsymbol{f} - \boldsymbol{n} \tag{3.4}$$

Unfortunately for this approach, there is no precise information on the noise $\boldsymbol{n}$ to compute this simple noise subtraction. Similarly, by standard linear algebra, one would be tempted to find the inverse of the linear operator $\boldsymbol{K}$ and obtain a clean image by,