

CHAPTER

5

**ATANGANA-BALEANU CASSON
HYBRID NANOFLUID
ON RIGA PLATE**

*Ridhwan Reyaz, Ahmad Qushairi Mohamad,
Noraihan Afiqah Rawi, and Wan Rukaida Wan Abdullah*

5.1 INTRODUCTION

Fluids are widely employed for transferring heat from one medium to another. Heat transfer applications for fluids are ubiquitous, spanning from electrical devices like refrigerators, computers, and air conditioning units to nuclear power plants, where they regulate heat in thermal reactors. The advent of nanofluids has significantly improved heat transfer properties in fluids. In the early 1990s, researchers discovered that introducing nanoparticles into a fluid can accelerate the rate of heat transmission (Choi & Eastman, 1995). Nanofluids typically consist of water, ethylene glycol, oil, copper, aluminium, ferromagnesium, and oxide metals as fluid bases, along with nanoparticles. There have been numerous studies on the boundary layer flow of nanofluids. For instance, Khalid et al. (2015) investigated the free convection flow of nanofluid with a ramping wall effect. They explored the behaviour of five different nanoparticles in a water-based fluid and solved the governing partial differential equations using the Laplace transform.

Analytical solutions were obtained to illustrate the temperature and velocity patterns of the nanofluid. On the other hand, Aly and Ebaid, (2016) conducted an analytical analysis of heat transfer rates in nanofluids, considering magnetohydrodynamic (MHD) and Marangoni radiation effects. Prasad et al. (2018) studied heat and mass transfer in copper and titanium oxide nanofluids with MHD and radiation absorption effects using water as the base fluid. Other researchers, such as Hussanan et al. (2019), Souayah et al. (2019), Uddin and Rasel, (2019), Mahanta et al. (2019), Krishna et al. (2021), Aleem et al. (2020), and Anwar et al. (2020), have also conducted analytical and numerical investigations on nanofluids.

Fractional derivatives introduce the concept of extending the traditional derivative to have an order of an arbitrary number or fraction. Over time, various definitions of fractional derivatives have been proposed, including Riemann-liouville, Hilfer, Caputo, Caputo-fabrizio, Atangana-baleanu, and others (Atangana, 2013; Luchko, 2020; Teodoro et al., 2019). Among these definitions, the Caputo-fabrizio fractional derivative (CFFD) stands out due to its non-singular kernel property, making it a favored choice. In contrast to the Caputo and Atangana-baleanu derivatives, the CFFD solution of a partial differential equation yields an integral function without involving any special functions. Although fractional derivatives have not yet found direct geometrical or physical representations in the realm of fluid mechanics, literature suggests that incorporating them into fluid mechanics models leads to unique yet viable solutions. As a result, researchers are likely to adopt these technologies to validate their findings in the near future, underscoring the significance of fractional derivatives in studying fluid flow behaviour.

The behaviour of boundary layers with fractional derivatives has been a subject of extensive research over the years. Khan et al. (2016) have been pioneers in studying analytical solutions for boundary layer flow using the Caputo derivative. Similar to the Caputo derivative, the Atangana-baleanu derivative is known for its non-singular kernels, leading to analytical solutions in the form of special functions such as the

Wright and Mittag-leffler functions. Due to these special functions, continuous solutions are sometimes impossible to calculate. To overcome this, a hybrid method of Laplace and Zakian's method of inverse Laplace transform has been explored.

Riga plates are flat plates with alternating arrangements of electrodes and magnets, commonly utilised as actuators to control fluid flow and reduce turbulence, especially in marine engineering applications for submarines and aquatic vessels. The presence of electrodes and magnets induces an electromagnetic current, resulting in an upthrust force known as the Lorentz force. This force can either facilitate or obstruct fluid flow depending on the Riga plate's location. Asogwa et al. (2022) conducted an analytical investigation on the impact of the Lorentz force from the Riga plate on Casson fluid flow with double convection. They also compared water-based nanofluids containing alumina-oxide and copper-oxide flowing vertically across a Riga plate, where copper-oxide proved to be a more efficient thermal conductor than alumina-oxide. Similar studies with comparable findings were conducted by Khatun et al. (2021) and others, such as Mallawi et al. (2021), Rasool et al. (2021), Bilal et al. (2021), and Nasrin et al. (2020).

To the best of authors' knowledge, there are limited studies on the analytical analysis of the impact a Riga plate on the free convection flow of Casson hybrid nanofluid with Caputo fractional derivative. Thus, the aim of this study is to investigate the effects of a hybrid nanofluid consisting of copper and alumina-oxide in a Carboxymethyl cellulose (CMC) base, using the Caputo fractional derivative, flowing freely over a vertical Riga plate.

5.2 CASSON HYBRID NANOFLUID FLOW OVER RIGA PLATE

The study investigates the behavior of a Casson hybrid nanofluid flowing unsteadily over an infinite Riga plate under the influence of free convection. The Riga plate is positioned vertically along the x-axis, with the y-axis perpendicular to it. As the nanofluid flows parallel to the x-axis