CHAPTER 5

Constructing Copula from a Bivariate Function

Ang Kah Yee, Rahmah Mohd Lokoman, and Siti Rohani Mohd Nor

5.1 INTRODUCTION

In the past years, several copula models have been developed and applied in hydrology (Bezak et al., 2018), climatology (Yu et al., 2014), banking industry (Domino et al., 2014, and Mensi et al., 2017) and finance (Lee et al., 2014; Cubillos-Rocha et al., 2019). Sklar (1959) introduces a copula to describe the dependence structure between the multivariate distribution functions combined with a univariate marginal distribution. However, instead of multivariate distribution functions, this chapter only focuses on the two distribution functions to construct a bivariate copula function: bivariate exponential function and bivariate quadratic function.

Several methods can be used to build the copula function by using the inversion method, generator function method, and algebraic method (Nelsen, 2006). However, in this chapter, the Rüschendorf method by Mah & Shitan (2014) is chosen to construct the copula function. It is because Rüschendorf offers many advantages since it involves simple mathematical integration computation and is easy to fit. Other than that, limited research has been done on constructing copula functions using the Rüschendorf method as far as the knowledge reaches. Therefore, this chapter aims to explore more on this technique to build the copula from the two bivariate functions.

This chapter is structured as follows. Section 5.2 outlines the construction of a bivariate copula model by using the Rüschendorf method. Section 5.3 presents and discusses the results of the proposed design model. Finally, Section 5.4 summarises the main conclusion of this chapter.

5.2 CONSTRUCTION OF COPULA

This section aims to build the bivariate copula function through the bivariate exponential function and bivariate quadratic function by using the Rüschendorf method. The steps involved in achieving this objective are shown in Algorithm 1.

Algorithm 1:

(a) Prove that the bivariate function is a joint probability density function.

(b) Compute the marginal functions, $f_x(x), f_y(y), A$, where x and y are random variables.

(c) Compute $f'(x, y) = f(x, y) - f_x(x) - f_y(y) + A$.

(d) Check that the two marginal functions and the unit square $f'(x, y)$ are zero.

(e) Set $g(x, y) = 1 + \theta f'(x, y)$, where θ is a copula parameter.

(f) Check that $g(x, y) > 0$.

(g) Construct $C(u, v)$, where u and v are the uniform random variables.

(h) Check the boundary conditions and 2-increasing properties of the copula function.

(i) Construct Kendall's tau and Spearman's rho.

(j) Determine the tail dependence.

The function is called the bivariate copula function if the two fundamental properties of the copula function are fulfilled (Nelsen, 2006), which are:

(1) Boundary Conditions:
$C(u,v)$ is the bivariate copula function. For all $u,v \in [0,1]$,

(a) $C(0,v)=0$

(b) $C(u,0)=0$

(c) $C(1,v)=v$

(d) $C(u,1)=u$

(e) $C(1,1)=1$ (5.1)

(2) 2-increasing property:

$$C(u_1,v_1)+C(u_2,v_2)-C(u_1,v_2)-C(u_2,v_1)\geq 0 \qquad (5.2)$$

for all $u_1,u_2,v_{1,}v_2 \in [0,1]$ and. $u_1 \leq u_2, v_1 \leq v_2$
The 2-increasing property is equivalent to $\frac{\partial^2 C(u,v)}{\partial u \partial v} \geq 0$ if $C(u,v)$ is differentiable two times (Trivedi & Zimmer, 2007).

5.2.1 Estimation of Copula Parameter

There are two bivariate functions used to build the bivariate copula; bivariate exponential function and bivariate quadratic function. The two functions respectively are presented as follows: