CHAPTER 8
Bivariate Copula Model for Rainfall Data

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8.1 INTRODUCTION

Copula functions are methods for modelling and building joint probability distributions and dependency structures. Copula is essential in integrating multivariate distributions based on their one-dimensional marginal distribution function. Copula functions may join multiple distributions into a single model in cases where dependencies between them are important. Copula modelling has been widely applied in a number of other fields, including survival analysis, actuarial science, and finance. Copulas have recently gained prominence for multivariate analysis in several fields, including financial studies by Frees and Valdez (1988) and Cherubini et al. (2004), flood flow analysis by Favre et al. (2004), Zhang and Singh (2006), Grimaldi and Serinaldi (2006), Genest and Favre (2007), Karmakar and Simonovic (2009), Klein et al. (2010), and Chowdhary et al. (2011), and for drought analysis by Kao and Govindaraju (2010) and Song and Singh (2010).

In hydrology, bivariate rainfall distribution has been modelled using copula (Zakaria et al., 2010). This model was performed at two locations in Australia's Murray-Darling Basin. Hume and Beechworth have been chosen as the locations. The findings show that the skew-*t* copula is appropriate for modelling monthly amounts of precipitation for corresponding stations in the Muray-Darling Basin. Therefore, the skew-*t* copula model is suggested in this study in order to model the monthly rainfall amounts at Kelantan river basin. Similar results may be obtained but it may provide information of other correlated stations at the river basin. Four copula families, named Clayton, Frank, Gumbel and Gaussian, are used in this paper for modelling precipitation from two selected meteorological stations in Kelantan: Stations 1 (located at Gua Musang) and Station 2 (located at Kuala Krai). The relationship between the two stations will be investigated, and marginal distributions for each variable will be established. These variables will be represented by a bivariate copula family.

8.2 STUDY REGION

Peninsular Malaysia is located on Malaysia's west coast and accounts for roughly 80% of the country's population and economy. In this chapter, we concentrated only on the eastern part of Peninsular Malaysia, specifically the Kelantan River Basin. The 44-year monthly rainfall data (1970–2014) that were obtained from the Malaysian Meteorological Department and the Malaysian Drainage and Irrigation Department for the two stations, Station 1 and Station 2, were analysed in this chapter. Figure 8.1 depicts a map of the research area.

Figure 8.1 Map of Eastern part of Peninsular Malaysia

8.3 METHODOLOGY

8.3.1 Fitting Distribution

The rainfall data distribution from both stations were modelled using the Generalized Extreme Value (GEV), Log Normal, Gamma, and Weibull distributions. The probability density function (PDF) and Cumulative Distribution Function (CDF) of these distributions are given in Table 8.1.

Table 8.1 PDF and CDF of the distribution function

Distribution	Distribution Function
Gamma α : shape β : scale γ : location	**PDF:** Three-Parameter distribution: $$f(v)=\frac{(v-\gamma)^{\alpha-1}}{\beta^{\alpha}\Gamma(\alpha)}\exp\left(-(v-\gamma)/\beta\right)$$ **CDF:** Three-Parameter distribution: $$F(v)=\frac{\Gamma_{\frac{(v-\gamma)}{\beta}}(\alpha)}{\Gamma(\alpha)}$$
GEV κ : shape σ : scale μ : location	**PDF:** For $k\neq 0$, $$f(v)=\frac{1}{\sigma}\exp\left(-(1+kz)^{-\frac{1}{k}}\right)(1+kz)^{-1-\frac{1}{k}}$$ For $k=0$ $$f(v)=\frac{1}{\sigma}\exp\left(-z-\exp(-z)\right)$$ **CDF:** For $k\neq 0$ $$F(v)=\exp\left(-(1+kz)^{-\frac{1}{k}}\right)$$ For $k=0$ $$F(v)=\exp\left(-\exp(-z)\right)$$ where $z\equiv\frac{v-\mu}{\sigma}$

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