

COPULA

MODELLING AND

ITS APPLICATION

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**Edited by
Fadhilah Yusof
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Preface

This edited book is a collection of studies, done over the years using the theory of copula. In this theory, the concept of dependency has been explored and applied in many applications, specifically in rainfall and drought analysis. The derivation of copula families is also discussed to emphasize on the mathematical foundations that exist in constructing the copula functions. This derivation is also showing the importance of the dependency of the marginal distributions to be known or measured to be incorporated in the copula functions. In estimating the parameters of the copula distributions, various methods of estimations are applied and compared in relation to the data used. Hence, the important roles in copula modelling in recent years are highlighted and the possible existence of relationship between variables of interest in the real application must be addressed appropriately.

This edited book can be used as a reference for students in undergraduate and postgraduate studies, who are interested in copula concept and modelling, especially the underlying mathematical formulations of the theory. Moreover, this book can also be useful to those researchers in applied fields who may need strong reference on the dependency theory or engineers in water resource management units who may need information on the copula theory in hydrological application.

Our special thanks go to the authors for their willingness to contribute to this edited book. These authors have one common interest that is using copula concepts in their research. The teamwork spirit among all the authors is commendable and that helps to smoothen the progress of this book. We are very

fortunate to have a good team of authors, not only from UTM, but also from UKM, UiTM and UMP. Excitingly, in this project we also have contributors from the Caraga University in Butuan, Philippines. This project will hopefully mark our initial effort in doing more collaborative work in the future.

Finally, we are grateful to all the Climate Change Research Group (CCRG) members, in particular to Dr Siti Rohani Mohd Nor, for her assistance in monitoring the progress of this edited book. We thank all for the continuous support and commitment and we are hopeful that there will be more edited books that may be published from CCRG in the future. Last but not least, our thanks and gratitude to the Department of Mathematical Sciences, UTM Centre for Industrial and Applied Mathematics (UTM-CIAM) and Faculty of Science for giving this opportunity to be the editors of this Edited Book.

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